

# Gravitational energy as dark energy: cosmic structure and apparent acceleration<sup>1</sup>

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**Abstract.** Below scales of about  $100h^{-1}\text{Mpc}$  our universe displays a complex inhomogeneous structure dominated by voids, with clusters of galaxies in sheets and filaments. The coincidence that cosmic expansion appears to start accelerating at the epoch when such structures form has prompted a number of researchers to question whether dark energy is a signature of a failure of the standard cosmology to properly account, on average, for the distribution of matter we observe. Here I discuss the timescape scenario, in which cosmic acceleration is understood as an apparent effect, due to gravitational energy gradients that grow when spatial curvature gradients become significant with the nonlinear growth of cosmic structure. I discuss conceptual issues related to the averaging problem, and their impact on the calibration of local geometry to the solutions of the volume-average evolution equations corrected by backreaction, and the question of nonbaryonic dark matter in the timescape framework. I further discuss recent work on defining observational tests for average geometric quantities which can distinguish the timescape model from a cosmological constant or other models of dark energy.

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## INTRODUCTION

This conference is titled a “Conference on Two Cosmological Models” but I think that what have been presented are certainly more than two cosmological models. From the point of view of proponents of the standard cosmology, the conference might seem to be dealing with too many cosmological models. The fact that there are a lot of ideas on the table is natural at any point in the history of science when observations present a fundamental crisis. We have reached such a point, given that our current standard cosmology only works by invoking unknown sources of “dark energy” and “dark matter”, which supposedly make up most of the stuff in the universe.

All of the models presented, including the standard  $\Lambda\text{CDM}$  cosmology, could be said to be relativistic in the sense that they obey either Einstein’s equations or some extension of Einstein gravity with a geometric diffeomorphism invariant action. What is at issue, however, is the manner in which different models seek to explain the observed large scale structure and motion of objects in the universe. Do we add new fields or modifications to the gravitational action, whose only influence is on cosmological scales, or do we seek to find deeper answers in the principles of general relativity?

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These are questions that Einstein struggled with when he first applied general relativity to cosmology [1]. He thought of spacetime as being a relational structure, and therefore the introduction of a cosmological constant – a vacuum energy in the fabric of space which made no direct connection to inertial properties of matter – was not a step he took lightly. I will take the viewpoint that rather than adding further epicycles to the gravitational action, the cosmological observations which we currently interpret in terms of dark energy are inviting us to think more deeply about the foundations of general relativity. There are questions in general relativity – relating to coarse-graining, averaging and the definition of energy in such contexts – which have never been fully resolved. These are the questions which I believe are of relevance to cosmology.

In this paper I will review the conceptual basis [2, 3, 4] and observational tests [5] of a cosmology model [2, 6], which represents a new approach to understanding the phenomenology of dark energy as a consequence of the effect of the growth of inhomogeneous structures. The basic idea, outlined in a nontechnical manner in ref. [7], is that as inhomogeneities grow one must consider not only their backreaction on average cosmic evolution, but also the variance in the geometry as it affects the calibration of clocks and rulers of ideal observers. Dark energy is then effectively realised as a misidentification of gravitational energy gradients.

Although the standard Lambda Cold Dark Matter ( $\Lambda$ CDM) model provides a good fit to many tests, there are tensions between some tests, and also a number of puzzles and anomalies. Furthermore, at the present epoch the observed universe is only statistically homogeneous once one samples on scales of 150–300 Mpc. Below such scales it displays a web-like structure, dominated in volume by voids. Some 40%–50% of the volume of the present epoch universe is in voids with  $\delta\rho/\rho \sim -1$  on scales of  $30h^{-1}$  Mpc [8], where  $h$  is the dimensionless parameter related to the Hubble constant by  $H_0 = 100h \text{ km sec}^{-1} \text{ Mpc}^{-1}$ . Once one also accounts for numerous minivoids, and perhaps also a few larger voids, then it appears that the present epoch universe is void-dominated. Clusters of galaxies are spread in sheets that surround these voids, and in thin filaments that thread them.

A number of different approaches have been taken to study inhomogeneous cosmologies. One large area of research is that of exact solutions of Einstein’s equations (see, e.g., ref. [9]), and of the Lemaître–Tolman–Bondi [10] (LTB) dust solution in particular. While one may mimic any luminosity distance relation with LTB models, generally the inhomogeneities required to match type Ia supernovae (SneIa) data are much larger than the typical scales of voids described above. Furthermore, one must assume the unlikely symmetry of a spherically symmetric universe about our point, which violates the Copernican principle. It is my view that while the LTB solutions are interesting toy models, one should retain the Copernican principle in a statistical sense, and one should seriously try to model the universe with those scales of inhomogeneity that we actually observe.

One particular consequence of a matter distribution that is only statistically homogeneous, rather than exactly homogeneous, is that when the Einstein equations are averaged they do not evolve as a smooth Friedmann–Lemaître–Robertson–Walker (FLRW) geometry. Instead the Friedmann equations are supplemented by additional backreaction

terms<sup>2</sup> [12]. Whether or not one can fully explain the expansion history of the universe as a consequence of the growth of inhomogeneities and backreaction, without a fluid-like dark energy, is the subject of ongoing debate [13].

A typical line of reasoning against backreaction is that of a plausibility argument [14]: if we *assume* a FLRW geometry with small perturbations, and estimate the magnitude of the perturbations from the typical rotational and peculiar velocities of galaxies, then the corrections of inhomogeneities are consistently small. This would be a powerful argument, were it not for the fact that at the present epoch galaxies are not homogeneously distributed. The Hubble Deep Field reveals that galaxy clusters were close to being homogeneous distributed at early epochs, but following the growth voids at redshifts  $z \lesssim 1$  that is no longer the case today. Therefore galaxies cannot be consistently treated as randomly distributed gas particles on the  $30h^{-1}$  Mpc scales [8] that dominate present cosmic structure below the scale of statistical homogeneity.

Over the past few years I have developed a new physical interpretation of cosmological solutions within the Buchert averaging scheme [2, 3, 6]. I start by noting that in the presence of strong spatial curvature gradients, not only should the average evolution equations be replaced by equations with terms involving backreaction, but the physical interpretation of average quantities must also account for the differences between the local geometry and the average geometry. In other words, geometric variance can be just as important as geometric averaging when it comes to the physical interpretation of the expansion history of the universe.

I proceed from the fact that structure formation provides a natural division of scales in the observed universe. As observers in galaxies, we and the objects we observe in other galaxies are necessarily in bound structures, which formed from density perturbations that were greater than critical density. If we consider the evidence of the large scale structure surveys on the other hand, then the average location by volume in the present epoch universe is in a void, which is negatively curved. We can expect systematic differences in spatial curvature between the average mass environment, in bound structures, and the volume-average environment, in voids.

Spatial curvature gradients will in general give rise to gravitational energy gradients, and herein lie the issues which I believe are key to understanding the phenomenon of dark energy. The definition of gravitational energy in general relativity is notoriously subtle. This is due to the equivalence principle, which means that we can always get rid of gravity near a point. As a consequence, the energy, momentum and angular momentum associated with the gravitational field, which have macroscopic effects on the relative calibrations of the clocks and rulers of observers, cannot be described by local quantities encoded in a fluid-like energy-momentum tensor. Instead they are at best *quasi-local* [15]. There is no general agreement on how to deal with quasi-local gravitational energy. It is my view that since the issue has its origin in the equivalence principle, we must return to first principles and reconsider the equivalence principle in the context of cosmological averages.

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<sup>2</sup> For a general review of averaging and backreaction see, e.g., ref. [11].

## THE COSMOLOGICAL EQUIVALENCE PRINCIPLE

In laying the foundations of general relativity, Einstein sought to refine our physical understanding of that most central physical concept: *inertia*. As he stated: “In a consistent theory of relativity there can be no inertia relatively to ‘space’, but only an inertia of masses relatively to one another” [1]. This is the general philosophy that underlies Mach’s principle, which strongly guided Einstein. However, the refinement of the understanding of inertia that Einstein left us with in relation to gravity, the Strong Equivalence Principle (SEP), only goes part-way in addressing Mach’s principle.

Mach’s principle may be stated [16, 17]: “*Local inertial frames (LIFs) are determined through the distributions of energy and momentum in the universe by some weighted average of the apparent motions*”. The SEP says nothing about the average effect of gravity, and therefore nothing about the suitable “weighted average of the apparent motions” of the matter in the universe. Since gravity for ordinary matter fields obeying the strong energy condition is universally attractive, the spacetime geometry of a universe containing matter is not stable, but is necessarily dynamically evolving. Therefore, accounting for the average effect of matter to address Mach’s principle means that any relevant frame in cosmological averages is one in which time symmetries of the Lorentz group in LIFs are removed.

My proposal for applying the equivalence principle on cosmological scales is to deal with the average effects of the evolving density by extending the SEP to larger regional frames while removing the time translation and boost symmetries of the LIF to define a *Cosmological Equivalence Principle* as follows [3]:

*At any event, always and everywhere, it is possible to choose a suitably defined space-time neighbourhood, the cosmological inertial frame (CIF), in which average motions (timelike and null) can be described by geodesics in a geometry that is Minkowski up to some time-dependent conformal transformation,*

$$ds_{\text{CIF}}^2 = a^2(\eta) [-d\eta^2 + dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)]. \quad (1)$$

Since the average geometry is a time-dependent conformal scaling of Minkowski space, the CEP reduces to the standard SEP if  $a(\eta)$  is constant, or alternatively over very short time intervals during which the time variation of  $a(\eta)$  can be neglected. The relation to cosmological averages is understood by the fact that (1) is the spatially flat FLRW metric. In the standard cosmology this is taken to be the geometry of the whole universe. Here, however, the whole universe is inhomogeneous while its geometry is restricted by the requirement that it is possible to always choose (1) as a regional average. This would rule out geometries with global anisotropies, such as Bianchi models, while hopefully leaving enough room to describe an inhomogeneous but statistically homogeneous universe like the one we observe.

To understand why an average geometry (1) is a relevant average reference geometry for the relative calibration of rulers and clocks in the absence of global Killing vectors, let us construct what I will call the *semi-tethered lattice* by the following means. Take a lattice of observers in Minkowski space, initially moving isotropically away from each nearest neighbour at uniform initial velocities. The lattice of observers are chosen to be equidistant along mutual oriented  $\hat{x}$ ,  $\hat{y}$  and  $\hat{z}$  axes. Now suppose that the observers are

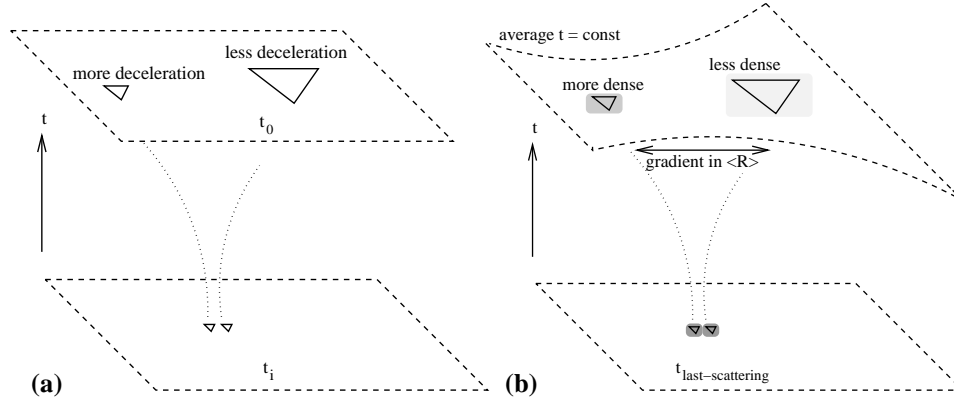
each connected to six others by tethers of negligible mass and identical tension along the mutually oriented spatial axes. The tethers are not fixed but unwind freely from spools on which an arbitrarily long supply of tether is wound. The tethers initially unreel at the same uniform rate, representing a “recession velocity”. Each observer carries synchronised clocks, and at a prearranged local proper time all observers apply brakes to each spool, the braking mechanisms having been pre-programmed to deliver the same impulse as a function of local time.

The semi-tethered lattice experiment is directly analogous to the decelerating volume expansion of (1) due to some average homogeneous matter density, because it maintains the homogeneity and isotropy of space over a region as large as the lattice. Work is done in applying the brakes, and energy can be extracted from this – just as kinetic energy of expansion of the universe is converted to other forms by gravitational collapse. Since brakes are applied in unison, however, there is *no net force on any observer in the lattice*, justifying the *inertial frame* interpretation, even though each observer has a nonzero 4-acceleration with respect to the global Minkowski frame. The braking function may have an arbitrary time profile; provided it is applied uniformly at every lattice site the clocks will remain synchronous in the comoving sense, as all observers have undergone the same relative deceleration.

Whereas the Strong Equivalence Principle leads us to define local inertial frames, related to each other by local Lorentz transformations acting at a point, the Cosmological Equivalence Principle refers to a *collective* symmetry of the background. In defining the averaging region of the CIF we are isolating just that part of the volume expansion which is regionally homogeneous and isotropic, and which is determined by the regionally homogeneous part of the background density.

Let us now consider two sets of disjoint semi-tethered lattices, with identical initial local expansion velocities, in a background static Minkowski space. (See Fig. 1(a).) Observers in the first congruence apply brakes in unison to decelerate homogeneously and isotropically at one rate. Observers in the second congruence do so similarly, but at a different rate. Suppose that when transformed to a global Minkowski frame, with time  $t$ , that at each time step the magnitudes of the 4-decelerations satisfy  $\alpha_1(t) > \alpha_2(t)$  for the respective congruences. By special relativity, since members of the first congruence decelerate more than those of the second congruence, at any time  $t$  their proper times satisfy  $\tau_1 < \tau_2$ . The members of the first congruence age less quickly than members of the second congruence.

By the CEP, the case of volume expansion of two disjoint regions of different average density in the actual universe is entirely analogous. The equivalence of the circumstance rests on the fact that the expansion of the universe was extremely uniform at the time of last scattering, by the evidence of the CMB. At that epoch all regions had almost the *same* density – with tiny fluctuations – and the same uniform Hubble flow. At late epochs, suppose that in the frame of any average cosmological observer there are expanding regions of *different* density which have decelerated by different amounts by a given time,  $t$ , according to that observer. Then by the CEP the local proper time of the comoving observers in the denser region, which has decelerated more, will be less than that of the equivalent observers in the less dense region which has decelerated less. (See Fig. 1(b).) Consequently the *proper time of the observers in the more dense CIF will be less than that of those in the less dense CIF*, by equivalence of the two situations.



**FIGURE 1.** Two equivalent situations: **(a)** in Minkowski space observers in separate semi-tethered lattices, initially expanding at the same rate, apply brakes homogeneously and isotropically within their respective regions but at different rates; **(b)** in the universe which is close to homogeneous and isotropic at last-scattering comoving observers in separated regions initially move away from each other isotropically, but experience different locally homogeneous isotropic decelerations as local density contrasts grow. In both cases there is a relative deceleration of the observer congruences and those in the region which has decelerated more will age less.

The fact that a global Minkowski observer does not exist in the second case does not invalidate the argument. The global Minkowski time is just a coordinate label. In the cosmological case the only restriction is that *the expansion of both average congruences must remain homogeneous and isotropic in local regions of different average density in the global average  $t = \text{const}$  slice*. Provided we can patch the regional frames together suitably, then if regions in such a slice *are still expanding* and have a significant density contrast we can expect a significant clock rate variance.

This equivalence directly establishes the idea of a *gravitational energy cost for a spatial curvature gradient*, since the existence of expanding regions of different density within an average  $t = \text{const}$  slice implies a gradient in the average Ricci scalar curvature,  $\langle \mathcal{R} \rangle$ , on one hand, while the fact that the local proper time varies on account of the relative deceleration implies a gradient in gravitational energy on the other.

In the actual universe, the question is: can the effect described above be significant enough to give a significant variation in the clocks of ideal isotropic observers (those who see an isotropic mean CMB) in regions of different density, who experience a relative deceleration of their regional volume expansions? Since we are dealing with weak fields the relative deceleration of the background is small. Nonetheless even if the relative deceleration is typically of order  $10^{-10} \text{ms}^{-2}$ , cumulatively over the age of the universe it leads to significant clock rate variances [3], of the order of 38%. Such a large effect is counterintuitive, as we are used to only considering time dilations due to relative accelerations within the static potentials of isolated systems. Essentially, we are dealing with a different physical effect concerning the relative synchronization of clocks in the absence of global Killing vectors. A small instantaneous relative deceleration can lead to cumulatively large differences, given one has the lifetime of the universe to play with. As a consequence the age of the universe itself becomes position-dependent. Since we and all the objects we observe are necessarily in regions of greater than critical density, on

account of structure formation we have a mass-biased view of the universe and cannot directly observe such variations.

In the standard ADM formalism one assumes the existence of a global rest frame comoving with the dust, and one makes a  $(3 + 1)$ -split of the Einstein equations from the point of view of fundamental observers who may be either comoving or tilted with respect to the dust. If the only symmetries that are allowed are diffeomorphisms of the global metric on one hand, and local Lorentz transformations corresponding to rotations and boosts on the other, then realistically there is no room within such an ADM formalism for clock rate variations of the order of magnitude dealt with in the timescape scenario [18]. However, such criticism overlooks the very real possibility the rest frame of dust is not globally defined, and furthermore it overlooks the crucial idea of regional averages introduced by the CEP. In proposing to separate the collective degree of freedom of the quasi-local regional volume expansion from other gravitational degrees of freedom, I am suggesting that we must consider average regional symmetries as a completely new ingredient in addition to the global diffeomorphisms and local Lorentz transformations with which we are familiar. This is a potential route to dealing with the unsolved problems of coarse-graining in general relativity.

At the epoch of last scattering dust may certainly be assumed to be atomic. However, once structures form geodesics cross and at the present epoch dust must be coarse-grained on at least the scale of galaxies in cosmological modelling. Thus the issue of the coarse-graining of dust is not merely a matter of choice, but of physical necessity if one is to consistently think about the interpretation of the Buchert formalism<sup>3</sup>. Of course, a detailed mathematical framework<sup>4</sup> for this still remains to be given in the timescape scenario. However, it is my view that mathematics is best guided by physical intuition rather than the reverse, and consequently my work to date has proceeded from making a phenomenological ansatz consistent with the CEP, to see whether the idea stands a chance of working.

## THE TIMESCAPE MODEL

I proceed from an ansatz that the variance in gravitational energy is correlated with the average spatial curvature in such a way as to implicitly solve the Sandage–de Vaucouleurs paradox that a statistically quiet, broadly isotropic, Hubble flow is observed deep below the scale of statistical homogeneity. In particular, galaxy peculiar velocities have a small magnitude with respect to a local regional volume expansion. Expanding regions of different densities are patched together so that the regionally measured expansion remains uniform. Such regional expansion refers to the variation of the regional

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<sup>3</sup> One can apply the Buchert formalism in a different manner – for example, to the problem of prescribed dust in exact solutions such as the LTB model [19]–[21] on globally well-defined spacelike hypersurfaces, where one specifically avoids solutions which develop vorticity or singularities. However, it is my view that to deal with the actual inhomogeneities of the observed universe then the average evolution of the Einstein equations should be regarded as a statistical description, and I approach the Buchert formalism in this sense.

<sup>4</sup> For one approach to coarse-graining, as opposed to averaging, see ref. [22].

proper length,  $\ell_r = \mathcal{V}^{1/3}$ , with respect to proper time of isotropic observers. Although voids open up faster, so that their proper volume increases more quickly, on account of gravitational energy gradients the local clocks will also tick faster in a compensating manner.

In order to deal with dust evolution from the surface of last scattering up to the present epoch, I assume that dust can be coarse-grained at the  $100h^{-1}\text{Mpc}$  scale of statistical homogeneity over which mass flows can be neglected. The manner in which I interpret the Buchert formalism is therefore different to that adopted by Buchert [12], who does not define the scale of coarse-graining of the dust explicitly. Details of the fitting of local observables to average quantities for solutions to the Buchert equations<sup>5</sup> are described in detail in refs. [2, 6]. Negatively curved voids, and spatially flat expanding wall regions within which galaxy clusters are located, are combined in a Buchert average

$$f_v(t) + f_w(t) = 1, \quad (2)$$

where  $f_w(t) = f_{wi}a_w^3/\bar{a}^3$  is the *wall volume fraction* and  $f_v(t) = f_{vi}a_v^3/\bar{a}^3$  is the *void volume fraction*,  $\mathcal{V} = \mathcal{V}_i\bar{a}^3$  being the present horizon volume, and  $f_{wi}$ ,  $f_{vi}$  and  $\mathcal{V}_i$  initial values at last scattering. The time parameter,  $t$ , is the volume-average time parameter of the Buchert formalism, but does not coincide with that of local measurements in galaxies. In trying to fit a FLRW solution to the universe we attempt to match our local spatially flat wall geometry

$$ds_{\bar{f}}^2 = -d\tau^2 + a_w^2(\tau) [d\eta_w^2 + \eta_w^2 d\Omega^2]. \quad (3)$$

to the whole universe, when in reality the calibration of rulers and clocks of ideal isotropic observers vary with gradients in spatial curvature and gravitational energy. By conformally matching radial null geodesics with those of the Buchert average solutions, the geometry (3) may be extended to cosmological scales as the dressed geometry

$$ds^2 = -d\tau^2 + a^2(\tau) [d\bar{\eta}^2 + r_w^2(\bar{\eta}, \tau) d\Omega^2] \quad (4)$$

where  $a = \bar{\gamma}^{-1}\bar{a}$ ,  $\bar{\gamma} = \frac{dt}{d\tau}$  is the relative lapse function<sup>6</sup> between wall clocks and volume-average ones,  $d\bar{\eta} = dt/\bar{a} = d\tau/a$ , and  $r_w = \bar{\gamma}(1 - f_v)^{1/3} f_{wi}^{-1/3} \eta_w(\bar{\eta}, \tau)$ , where  $\eta_w$  is given by integrating  $d\eta_w = f_{wi}^{1/3} d\bar{\eta} / [\bar{\gamma}(1 - f_v)^{1/3}]$  along null geodesics.

In addition to the bare cosmological parameters which describe the Buchert equations, one obtains dressed parameters relative to the geometry (4). For example, the dressed matter density parameter is  $\Omega_M = \bar{\gamma}^3 \bar{\Omega}_M$ , where  $\bar{\Omega}_M = 8\pi G \bar{\rho}_{M0} \bar{a}_0^3 / (3\bar{H}^2 \bar{a}^3)$  is the bare

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<sup>5</sup> The model of Wiegand and Buchert [23], briefly described by Buchert in the present volume [24], has similarities to the present model but also differs from it in certain key aspects. In particular, (i) the observational interpretation of the Buchert averages is different; (ii) the walls and voids are taken to have internal backreaction in the case or refs. [23, 24] but not here; and (iii) the interpretation of the initial wall and void fractions at last scattering is different. In respect of the last point, since walls and voids do not exist at the surface of last scattering, I take the view that the vast bulk of the present horizon volume that averages to critical density gives  $f_{wi} \simeq 1$ , while  $f_{vi} = 1 - f_{wi}$  is the small positive fraction of the present horizon volume that consists of uncompensated underdense regions at last scattering surface.

<sup>6</sup> This is a phenomenological function rather than the lapse function prescribed by the ADM formalism.



matter density parameter. The dressed parameters take numerical values close to the ones inferred in standard FLRW models.

## Apparent acceleration and Hubble flow variance

The gradient in gravitational energy and cumulative differences of clock rates between wall observers and volume average ones has important physical consequences. Using the exact solution obtained in ref. [6], one finds that a volume average observer would infer an effective deceleration parameter  $\bar{q} = -\ddot{a}/(\bar{H}^2 \bar{a}) = 2(1 - f_v)^2 / (2 + f_v)^2$ , which is always positive since there is no global acceleration. However, a wall observer infers a dressed deceleration parameter

$$q = \frac{-1}{H^2 a} \frac{d^2 a}{d\tau^2} = \frac{-(1 - f_v)(8f_v^3 + 39f_v^2 - 12f_v - 8)}{(4 + f_v + 4f_v^2)^2}, \quad (5)$$

where the dressed Hubble parameter is given by

$$H = a^{-1} \frac{d}{d\tau} a = \bar{\gamma} \bar{H} - \dot{\bar{\gamma}} = \bar{\gamma} \bar{H} - \bar{\gamma}^{-1} \frac{d}{d\tau} \bar{\gamma}. \quad (6)$$

At early times when  $f_v \rightarrow 0$  the dressed and bare deceleration parameter both take the Einstein–de Sitter value  $q \simeq \bar{q} \simeq \frac{1}{2}$ . However, unlike the bare parameter which monotonically decreases to zero, the dressed parameter becomes negative when  $f_v \simeq 0.59$  and  $\bar{q} \rightarrow 0^-$  at late times. For the best-fit parameters<sup>7</sup> the apparent acceleration begins at a redshift  $z \simeq 0.9$ .

Cosmic acceleration is thus revealed as an apparent effect which arises due to the cumulative clock rate variance of wall observers relative to volume–average observers. It becomes significant only when the voids begin to dominate the universe by volume. Since the epoch of onset of apparent acceleration is directly related to the void fraction,  $f_v$ , this solves one cosmic coincidence problem.

In addition to apparent cosmic acceleration, a second important apparent effect will arise if one considers scales below that of statistical homogeneity. By any one set of clocks it will appear that voids expand faster than wall regions. Thus a wall observer will see galaxies on the far side of a dominant void of diameter  $30h^{-1}$  Mpc recede at a rate greater than the dressed global average  $H_0$ , while galaxies within an ideal wall will recede at a rate less than  $H_0$ . Since the bare Hubble parameter  $\bar{H}$  provides a measure of the uniform quasi-local flow, it must also be the “local value” within an ideal wall at any epoch; i.e., eq. (6) gives a measure of the variance in the apparent Hubble flow. The best-fit parameters [25] give a dressed Hubble constant  $H_0 = 61.7_{-1.1}^{+1.2} \text{ km sec}^{-1} \text{ Mpc}^{-1}$ , and a bare Hubble constant  $\bar{H}_0 = 48.2_{-2.4}^{+2.0} \text{ km sec}^{-1} \text{ Mpc}^{-1}$ . The present epoch variance is 17–22%.

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<sup>7</sup> Here I will simply adopt the parameters found in ref. [25] from a fit to the Riess07 gold dataset [26]. A more recent analysis [27] shows that the best-fit parameters are sensitive to the method of supernova data reduction, and unknown systematic issues remain to be resolved. The parameters determined from the Riess07 dataset are in the mid-range of those determined by MLCS methods from larger datasets [27].

Since voids dominate the universe by volume at the present epoch, any observer in a galaxy in a typical wall region will measure locally higher values of the Hubble constant, with peak values of order  $72 \text{ km sec}^{-1} \text{ Mpc}^{-1}$  at the  $30h^{-1} \text{ Mpc}$  scale of the dominant voids. Over larger distances, as the line of sight intersects more walls as well as voids, a radial spherically symmetric average will give an average Hubble constant whose value decreases from the maximum at the  $30h^{-1} \text{ Mpc}$  scale to the dressed global average value, as the scale of homogeneity is approached at roughly the baryon acoustic oscillation (BAO) scale of  $110h^{-1} \text{ Mpc}$ . This predicted effect could account for the Hubble bubble [28] and more detailed studies of the scale dependence of the local Hubble flow [29].

In fact, the variance of the local Hubble flow below the scale of homogeneity should correlate strongly to observed structures in a manner which has no equivalent prediction in FLRW models.

There is already evidence from the study of large-scale bulk flows that apparent peculiar velocities determined in the FLRW framework have a magnitude in excess of the expectations of the standard  $\Lambda\text{CDM}$  model [30, 31]. In the present framework, rather than having a uniform expansion (with respect to one set of clocks), with respect to which peculiar velocities are defined, we have variations in the expansion rate in regions of different density which are expanding but decelerating at different rates. Nonetheless, given that our location is on the edge of a dominant void and a wall [32] the equivalent maximum peculiar velocity can be estimated as

$$v_{\text{pec}} = \left(\frac{3}{2}\bar{H}_0 - H_0\right) \frac{30}{h} \text{ Mpc} = 510_{-260}^{+210} \text{ km/s} \quad (7)$$

assuming a diameter of  $30h^{-1} \text{ Mpc}$  for the local dominant void. This rough estimate is of a magnitude consistent with observation.

## FUTURE OBSERVATIONAL TESTS

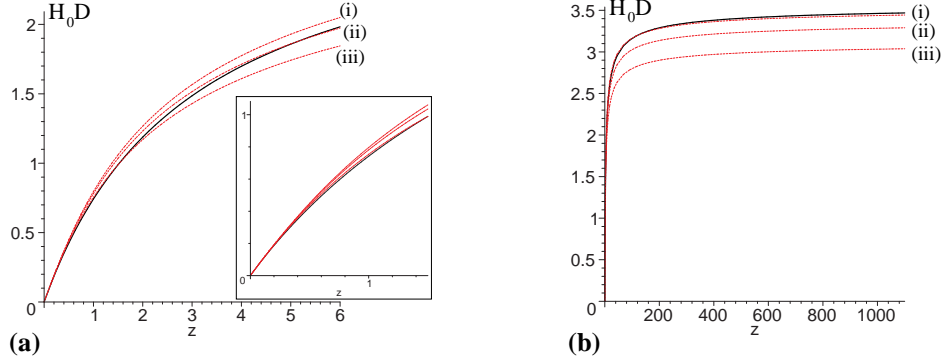
There are two types of potential cosmological tests that can be developed; those relating to scales below that of statistical homogeneity as discussed above, and those that relate to averages on our past light cone on scales much greater than the scale of statistical homogeneity. The second class of tests includes equivalents to all the standard cosmological tests of the standard FLRW model with Newtonian perturbations. This second class of tests can be further divided into tests which just deal with the bulk cosmological averages (luminosity and angular diameter distances etc), and those that deal with the variance from the growth of structures (late epoch integrated Sachs–Wolfe effect, cosmic shear, redshift space distortions etc). Here I will concentrate solely on the simplest tests which are directly related to luminosity and angular diameter distance measures.

In the timescape cosmology we have an effective dressed luminosity distance

$$d_L = a_0(1+z)r_w, \quad (8)$$

where  $a_0 = \bar{\gamma}_0^{-1}\bar{a}_0$ , and

$$r_w = \bar{\gamma}(1-f_v)^{1/3} \int_t^{t_0} \frac{dt'}{\bar{\gamma}(t')(1-f_v(t'))^{1/3}\bar{a}(t')}. \quad (9)$$



**FIGURE 2.** The effective comoving distance  $H_0 D(z)$  is plotted for the best-fit timescape (TS) model, with  $f_{v0} = 0.762$ , (solid line); and for various spatially flat  $\Lambda$ CDM models (dashed lines). The parameters for the dashed lines are (i)  $\Omega_{M0} = 0.249$  (best-fit to WMAP5 only [33]); (ii)  $\Omega_{M0} = 0.279$  (joint best-fit to Snela, BAO and WMAP5); (iii)  $\Omega_{M0} = 0.34$  (best-fit to Riess07 Snela only [26]). Panel (a) shows the redshift range  $z < 6$ , with an inset for  $z < 1.5$ , which is the range tested by current Snela data. Panel (b) shows the range  $z < 1100$  up to the surface of last scattering, tested by WMAP.

We can also define an *effective angular diameter distance*,  $d_A$ , and an *effective comoving distance*,  $D$ , to a redshift  $z$  in the standard fashion

$$d_A = \frac{D}{1+z} = \frac{d_L}{(1+z)^2}. \quad (10)$$

A direct method of comparing the distance measures with those of homogeneous models with dark energy, is to observe that for a standard spatially flat cosmology with dark energy obeying an equation of state  $P_D = w(z)\rho_D$ , the quantity

$$H_0 D = \int_0^z \frac{dz'}{\sqrt{\Omega_{M0}(1+z')^3 + \Omega_{D0} \exp \left[ 3 \int_0^{z'} \frac{(1+w(z''))}{1+z''} dz'' \right]}}, \quad (11)$$

does not depend on the value of the Hubble constant,  $H_0$ , but only directly on  $\Omega_{M0} = 1 - \Omega_{D0}$ . Since the best-fit values of  $H_0$  are potentially different for the different scenarios, a comparison of  $H_0 D$  curves as a function of redshift for the timescape model versus the  $\Lambda$ CDM model gives a good indication of where the largest differences can be expected, independently of the value of  $H_0$ . Such a comparison is made in Fig. 2.

We see that as redshift increases the timescape model interpolates between  $\Lambda$ CDM models with different values of  $\Omega_{M0}$ . For redshifts  $z \lesssim 1.5$   $D_{\text{TS}}$  is very close to  $D_{\Lambda\text{CDM}}$  for the parameter values  $(\Omega_{M0}, \Omega_{\Lambda0}) = (0.34, 0.66)$  (model (iii)) which best-fit the Riess07 supernovae (Snela) data [26] only, by our own analysis. For very large redshifts that approach the surface of last scattering,  $z \lesssim 1100$ , on the other hand,  $D_{\text{TS}}$  very closely matches  $D_{\Lambda\text{CDM}}$  for the parameter values  $(\Omega_{M0}, \Omega_{\Lambda0}) = (0.249, 0.751)$  (model (i)) which best-fit WMAP5 only [33]. Over redshifts  $2 \lesssim z \lesssim 10$ , at which scales independent tests are conceivable,  $D_{\text{TS}}$  makes a transition over corresponding curves of  $D_{\Lambda\text{CDM}}$  with intermediate values of  $(\Omega_{M0}, \Omega_{\Lambda0})$ . The  $D_{\Lambda\text{CDM}}$  curve for joint best-fit parameters to Snela, BAO measurements and WMAP5 [33],  $(\Omega_{M0}, \Omega_{\Lambda0}) = (0.279, 0.721)$  is best-matched over the range  $5 \lesssim z \lesssim 6$ , for example.

The difference of  $D_{\text{TS}}$  from any single  $D_{\Lambda\text{CDM}}$  curve is perhaps most pronounced in the range  $2 \lesssim z \lesssim 6$ , which may be an optimal regime to probe in future experiments. Gamma-ray bursters (GRBs) now probe distances to redshifts  $z \lesssim 8.3$ , and could be very useful if their properties could be understood to the extent that they might be reliably used as standard candles. A considerable amount of work has already been done on Hubble diagrams for GRBs. (See, e.g., [34].) Much more work is needed to nail down systematic uncertainties, but GRBs may eventually provide a definitive test in future. An analysis of the timescape model Hubble diagram using 69 GRBs has just been performed by Schaefer [35], who finds that the timescape model fits the data better than the concordance  $\Lambda\text{CDM}$  model, but not yet by a huge margin<sup>8</sup>. As more data is accumulated, it should become possible to distinguish the models if the issues with the standardization of GRBs can be ironed out.

### The effective “equation of state”

The shape of the  $H_0 D$  curves depicted in Fig. 2 represents the observable quantity one is actually measuring in tests some researchers loosely refer to as “measuring the equation of state”. For spatially flat dark energy models, with  $H_0 D$  given by (11), one finds that the function  $w(z)$  appearing in the fluid equation of state  $P_D = w(z)\rho_D$  is related to the first and second derivatives of (11) by

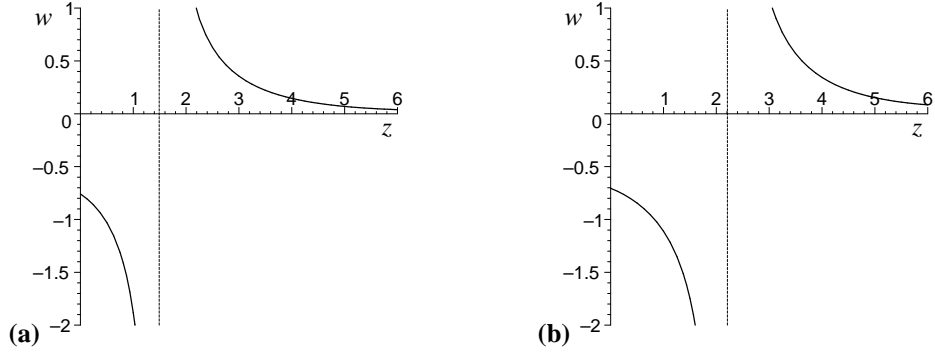
$$w(z) = \frac{\frac{2}{3}(1+z)D'^{-1}D'' + 1}{\Omega_{M0}(1+z)^3 H_0^2 D'^2 - 1} \quad (12)$$

where prime denotes a derivative with respect to  $z$ . Such a relation can be applied to observed distance measurements, regardless of whether the underlying cosmology has dark energy or not. Since it involves first and second derivatives of the observed quantities, it is actually much more difficult to determine observationally than directly fitting  $H_0 D(z)$ .

The equivalent of the “equation of state”,  $w(z)$ , for the timescape model is plotted in Fig. 3. The fact that  $w(z)$  is undefined at a particular redshift and changes sign through  $\pm\infty$  simply reflects the fact that in (12) we are dividing by a quantity which goes to zero for the timescape model, even though the underlying curve of Fig. 2 is smooth. Since one is not dealing with a dark energy fluid in the present case,  $w(z)$  simply has no physical meaning. Nonetheless, phenomenologically the results do agree with the usual inferences about  $w(z)$  for fits of standard dark energy cosmologies to SNeIa data. For the canonical model of Fig. 3(a) one finds that the average value of  $w(z) \simeq -1$  on the range  $z \lesssim 0.7$ , while the average value of  $w(z) < -1$  if the range of redshifts is extended to higher values. The  $w = -1$  “phantom divide” is crossed at  $z \simeq 0.46$  for  $f_{v0} \simeq 0.76$ . One recent study [37] finds mild 95% evidence for an equation of state that crosses the phantom divide from  $w > -1$  to  $w < -1$  in the range  $0.25 < z < 0.75$  in accord with the

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<sup>8</sup> By contrast the conformal gravity model of Mannheim [36] produced a worse fit, while the Chaplyagin gas fit best only in the limit that its parameters reduce to those of the  $\Lambda\text{CDM}$  model [35].



**FIGURE 3.** The artificial equivalent of an equation of state constructed using the effective comoving distance (12), plotted for the timescape tracker solution with best-fit value  $f_{v0} = 0.762$ , and two different values of  $\Omega_{M0}$ : **(a)** the canonical dressed value  $\Omega_{M0} = \frac{1}{2}(1 - f_{v0})(2 + f_{v0}) = 0.33$ ; **(b)**  $\Omega_{M0} = 0.279$ .

timescape expectation. By contrast, another study [38] at redshifts  $z < 1$  draws different conclusions about dynamical dark energy, but for the given uncertainties in  $w(z)$  the data is consistent with Fig. 2(a) as well as with a cosmological constant [5].

The fact that  $w(z)$  is a different sign to the dark energy case for  $z > 2$  is another way of viewing our statement above that the redshift range  $2 \lesssim z \lesssim 6$  may be optimal for discriminating model differences.

### The $H(z)$ measure

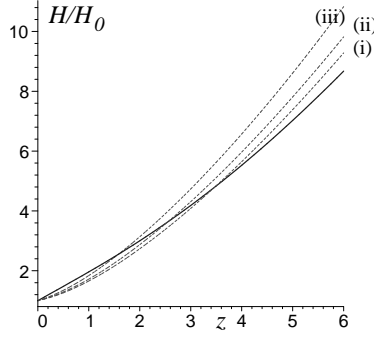
Further observational diagnostics can be devised if the expansion rate  $H(z)$  can be observationally determined as a function of redshift. Recently such a determination of  $H(z)$  at  $z = 0.24$  and  $z = 0.43$  has been made using redshift space distortions of the BAO scale in the  $\Lambda$ CDM model [39]. This technique is of course model dependent, and the Kaiser effect would have to be re-examined in the timescape model before a direct comparison of observational results could be made. A model-independent measure of  $H(z)$ , the redshift time drift test, is discussed below.

In Fig. 4 we compare  $H(z)/H_0$  for the timescape model to spatially flat  $\Lambda$ CDM models with the same parameters chosen in Fig. 2. The most notable feature is that the slope of  $H(z)/H_0$  is less than in the  $\Lambda$ CDM case, as is to be expected for a model whose (dressed) deceleration parameter varies more slowly than for  $\Lambda$ CDM.

### The $Om(z)$ measure

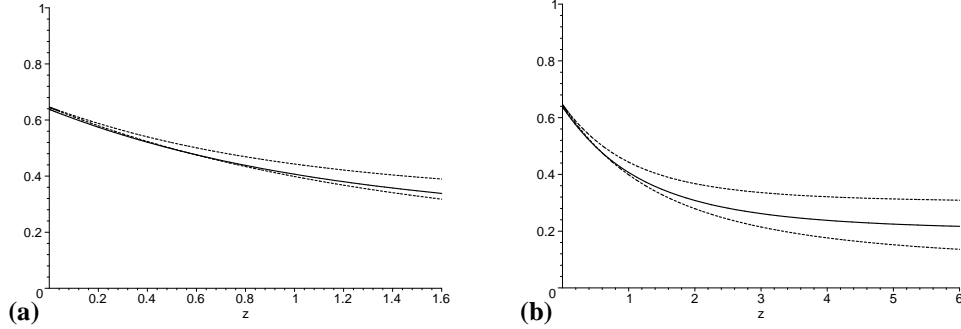
Recently a number of authors [40, 41, 42] have discussed various roughly equivalent diagnostics of dark energy. For example, Sahni, Shafieloo and Starobinsky [41], have proposed a diagnostic function

$$Om(z) = \left[ \frac{H^2(z)}{H_0^2} - 1 \right] [(1+z)^3 - 1]^{-1}, \quad (13)$$



**FIGURE 4.** The function  $H_0^{-1}H(z)$  for the timescape model with  $f_{v0} = 0.762$  (solid line) is compared to  $H_0^{-1}H(z)$  for three spatially flat  $\Lambda$ CDM models with the same values of  $(\Omega_{M0}, \Omega_{\Lambda0})$  as in Fig. 2 (dashed lines).

on account of the fact that it is equal to the constant present epoch matter density parameter,  $\Omega_{M0}$ , at all redshifts for a spatially flat FLRW model with pressureless dust and a cosmological constant. However, it is not constant if the cosmological constant is replaced by other forms of dark energy. For general FLRW models,  $H(z) = [D'(z)]^{-1} \sqrt{1 + \Omega_{k0} H_0^2 D^2(z)}$ , which only involves a single derivatives of  $D(z)$ . Thus the diagnostic (13) is easier to reconstruct observationally than the equation of state parameter,  $w(z)$ .

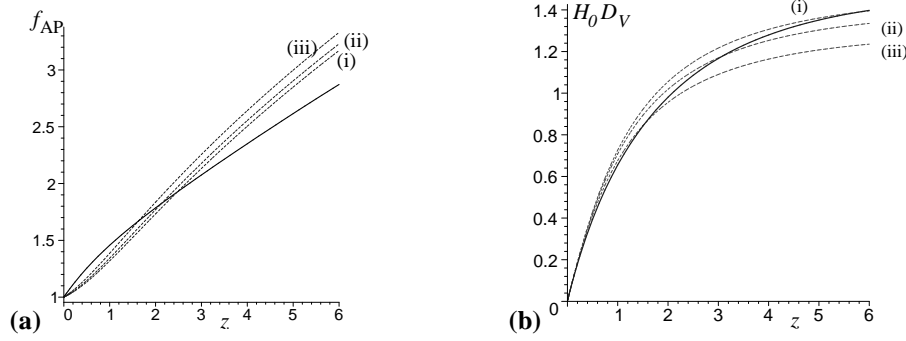


**FIGURE 5.** The dark energy diagnostic  $Om(z)$  of Sahni, Shafieloo and Starobinsky [41] plotted for the timescape tracker solution with best-fit value  $f_{v0} = 0.762$  (solid line), and  $1\sigma$  limits (dashed lines) from ref. [25]: (a) for the redshift range  $0 < z < 1.6$  as shown in ref. [43]; (b) for the redshift range  $0 < z < 6$ .

The quantity  $Om(z)$  is readily calculated for the timescape model, and the result is displayed in Fig. 5. What is striking about Fig. 5, as compared to the curves for quintessence and phantom dark energy models as plotted in ref. [41], is that the initial value

$$Om(0) = \frac{2}{3} H'|_0 = \frac{2(8f_{v0}^3 - 3f_{v0}^2 + 4)(2 + f_{v0})}{(4f_{v0}^2 + f_{v0} + 4)^2} \quad (14)$$

is substantially larger than in the spatially flat dark energy models. Furthermore, for the timescape model  $Om(z)$  does not asymptote to the dressed density parameter  $\Omega_{M0}$  in any redshift range. For quintessence models  $Om(z) > \Omega_{M0}$ , while for phantom models  $Om(z) < \Omega_{M0}$ , and in both cases  $Om(z) \rightarrow \Omega_{M0}$  as  $z \rightarrow \infty$ . In the timescape model,



**FIGURE 6.** (a) The Alcock–Paczyński test function  $f_{\text{AP}} = HD/z$ ; and (b) the BAO radial test function  $H_0 D_V = H_0 D f_{\text{AP}}^{-1/3}$ . In each case the timescape model with  $f_{v0} = 0.762$  (solid line) is compared to three spatially flat  $\Lambda$ CDM models with the same values of  $(\Omega_{M0}, \Omega_{\Lambda0})$  as in Fig. 2 (dashed lines).

$Om(z) > \Omega_{M0} \simeq 0.33$  for  $z \lesssim 1.7$ , while  $Om(z) < \Omega_{M0}$  for  $z \gtrsim 1.7$ . It thus behaves more like a quintessence model for low  $z$ , in accordance with Fig. 3. However, the steeper slope and the different large  $z$  behaviour mean the diagnostic is generally very different to that of typical dark energy models. For large  $z$ ,  $\bar{\Omega}_{M0} < Om(\infty) < \Omega_{M0}$ , if  $f_{v0} > 0.25$ .

Interestingly enough, a recent analysis of SNeIa, BAO and CMB data [43] for dark energy models with two different empirical fitting functions for  $w(z)$  gives an intercept  $Om(0)$  which is larger than expected for typical quintessence or phantom energy models, and in the better fit of the two models the intercept (see Fig. 3 of ref. [43]) is close to the value expected for the timescape model, which is tightly constrained to the range  $0.638 < Om(0) < 0.646$  if  $f_{v0} = 0.76^{+0.12}_{-0.09}$ .

## The Alcock–Paczyński test and baryon acoustic oscillations

Some time ago Alcock and Paczyński devised a test [44] which relies on comparing the radial and transverse proper length scales of spherical standard volumes comoving with the Hubble flow. This test, which determines the function

$$f_{\text{AP}} = \frac{1}{z} \left| \frac{\delta\theta}{\delta z} \right| = \frac{HD}{z}, \quad (15)$$

was originally conceived to distinguish FLRW models with a cosmological constant from those without a  $\Lambda$  term. The test is free from many evolutionary effects, but relies on one being able to remove systematic distortions due to peculiar velocities.

Current detections of the BAO scale in galaxy clustering statistics [45, 46] can in fact be viewed as a variant of the Alcock–Paczyński test, as they make use of both the transverse and radial dilations of the fiducial comoving BAO scale to present a measure

$$D_V = \left[ \frac{zD^2}{H(z)} \right]^{1/3} = D f_{\text{AP}}^{-1/3}. \quad (16)$$

In Fig. 6 the Alcock–Paczyński test function (15) and BAO scale measure (16) of the timescape model are compared to those of the spatially flat  $\Lambda$ CDM model with different values of  $(\Omega_{\Lambda 0}, \Omega_{\Lambda 0})$ . Over the range of redshifts  $z < 1$  studied currently with galaxy clustering statistics, the  $f_{\text{AP}}$  curve distinguishes the timescape model from the  $\Lambda$ CDM models much more strongly than the  $D_V$  test function. In particular, the timescape  $f_{\text{AP}}$  has a distinctly different shape to that of the  $\Lambda$ CDM model, being convex. The primary reason for use of the integral measure (16) has been a lack of data. Future measurements with enough data to separate the radial and angular BAO scales are a potentially powerful way of distinguishing the timescape model from  $\Lambda$ CDM.

Recently Gaztañaga, Cabré and Hui [39] have made the first efforts to separate the radial and angular BAO scales in different redshift slices. Although they have not yet published separate values for the radial and angular scales, their results are interesting when compared to the expectations of the timescape model. Their study yields best-fit values of the present total matter and baryonic matter density parameters,  $\Omega_{M0}$  and  $\Omega_{B0}$ , which are in tension with WMAP5 parameters fit to the  $\Lambda$ CDM model. In particular, the ratio of nonbaryonic cold dark matter to baryonic matter has a best-fit value  $\Omega_{C0}/\Omega_{B0} = (\Omega_{M0} - \Omega_{B0})/\Omega_{B0}$  of 3.7 in the  $0.15 < z < 0.3$  sample, 2.6 in the  $0.4 < z < 0.47$  sample, and 3.6 in the whole sample, as compared to the expected value of 6.1 from WMAP5. The analysis of the 3-point correlation function yields similar conclusions, with a best fit [47]  $\Omega_{M0} = 0.28 \pm 0.05$ ,  $\Omega_{B0} = 0.079 \pm 0.025$ . By comparison, the parameter fit to the timescape model of ref. [25] yields dressed parameters  $\Omega_{M0} = 0.33^{+0.11}_{-0.16}$ ,  $\Omega_{B0} = 0.080^{+0.021}_{-0.013}$ , and a ratio  $\Omega_{C0}/\Omega_{B0} = 3.1^{+2.5}_{-2.4}$ . Since homogeneous dark energy models are not generally expected to give rise to a renormalization of the ratio of nonbaryonic to baryonic matter, this is encouraging for the timescape model.

## Test of (in)homogeneity

Recently Clarkson, Bassett and Lu [48] have constructed what they call a “test of the Copernican principle” based on the observation that for homogeneous, isotropic models which obey the Friedmann equation, the present epoch curvature parameter, a constant, may be written as

$$\Omega_{k0} = \frac{[H(z)D'(z)]^2 - 1}{[H_0 D(z)]^2} \quad (17)$$

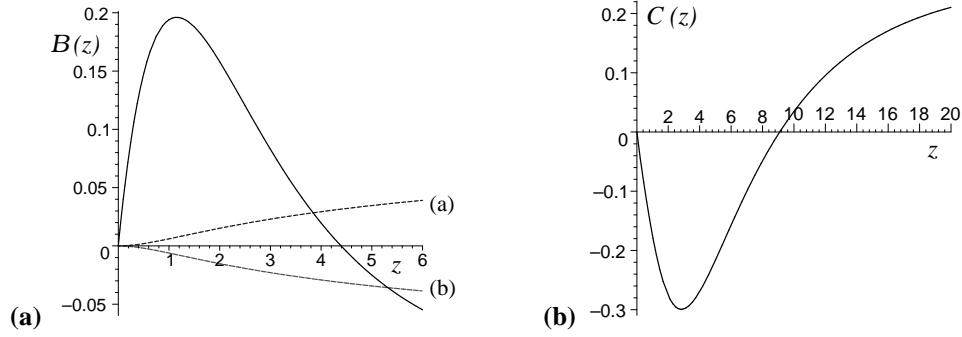
for all  $z$ , irrespective of the dark energy model or any other model parameters. Consequently, taking a further derivative, the quantity

$$\mathcal{C}(z) \equiv 1 + H^2(DD'' - D'^2) + HH'DD' \quad (18)$$

must be zero for all redshifts for any FLRW geometry.

A deviation of  $\mathcal{C}(z)$  from zero, or of (17) from a constant value, would therefore mean that the assumption of homogeneity is violated. Although this only constitutes a test of the assumption of the Friedmann equation, i.e., of the Cosmological Principle rather than the broader Copernican Principle adopted in ref. [2], the average inhomogeneity will give a clear and distinct prediction of a nonzero  $\mathcal{C}(z)$  for the timescape model.





**FIGURE 7. Left panel:** The (in)homogeneity test function  $\mathcal{B}(z) = [HD']^2 - 1$  is plotted for the timescape tracker solution with best-fit value  $f_{v0} = 0.762$  (solid line), and compared to the equivalent curves  $\mathcal{B} = \Omega_{k0}(H_0 D)^2$  for two different  $\Lambda$ CDM models with small curvature: **(a)**  $\Omega_{M0} = 0.28, \Omega_{\Lambda0} = 0.71, \Omega_{k0} = 0.01$ ; **(b)**  $\Omega_{M0} = 0.28, \Omega_{\Lambda0} = 0.73, \Omega_{k0} = -0.01$ . **Right panel:** The (in)homogeneity test function  $\mathcal{C}(z)$  is plotted for the  $f_{v0} = 0.762$  tracker solution.

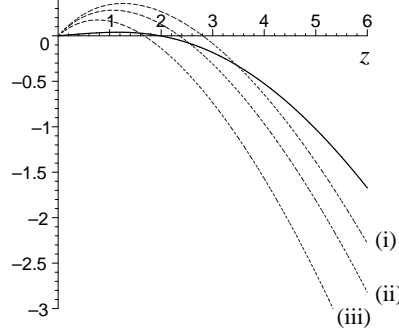
The functions (17) and (18) are computed in ref. [5]. Observationally it is more feasible to fit (17) which involves one derivative less of redshift. In Fig. 7 we exhibit both  $\mathcal{C}(z)$ , and also the function  $\mathcal{B}(z) = [HD']^2 - 1$  from the numerator of (17) for the timescape model, as compared to two  $\Lambda$ CDM models with a small amount of spatial curvature. A spatially flat FLRW model would have  $\mathcal{B}(z) \equiv 0$ . In other FLRW cases  $\mathcal{B}(z)$  is always a monotonic function whose sign is determined by that of  $\Omega_{k0}$ . An open  $\Lambda = 0$  universe with the same  $\Omega_{M0}$  would have a monotonic function  $\mathcal{B}(z)$  very much greater than that of the timescape model.

## Time drift of cosmological redshifts

For the purpose of the  $Om(z)$  and (in)homogeneity tests considered in the last section,  $H(z)$  must be observationally determined, and this is difficult to achieve in a model-independent way. There is one way of achieving this, however, namely by measuring the time variation of the redshifts of different sources over a sufficiently long time interval [49], as has been discussed recently by Uzan, Clarkson and Ellis [50]. Although the measurement is extremely challenging, it may be feasible over a 20 year period by precision measurements of the Lyman- $\alpha$  forest in the redshift range  $2 < z < 5$  with the next generation of Extremely Large Telescopes [51].

In ref. [5] an analytic expression for  $H_0^{-1} \frac{dz}{d\tau}$  is determined, the derivative being with respect to wall time for observers in galaxies. The resulting function is displayed in Fig. 8 for the best-fit timescape model with  $f_{v0} = 0.762$ , where it is compared to the equivalent function for three different spatially flat  $\Lambda$ CDM models. What is notable is that the curve for the timescape model is considerably flatter than those of the  $\Lambda$ CDM models. This may be understood to arise from the fact that the magnitude of the apparent acceleration is considerably smaller in the timescape model, as compared to the magnitude of the acceleration in  $\Lambda$ CDM models. For models in which there is no apparent acceleration whatsoever, one finds that  $H_0^{-1} \frac{dz}{d\tau}$  is always negative. If there is cosmic acceleration at

late epochs, real or apparent, then  $H_0^{-1} \frac{dz}{d\tau}$  will become positive at low redshifts, though at a somewhat larger redshift than that at which acceleration is deemed to have begun.



**FIGURE 8.** The function  $H_0^{-1} \frac{dz}{d\tau}$  for the timescape model with  $f_{v0} = 0.762$  (solid line) is compared to  $H_0^{-1} \frac{dz}{d\tau}$  for three spatially flat  $\Lambda$ CDM models with the same values of  $(\Omega_{M0}, \Omega_{\Lambda0})$  as in Fig. 2 (dashed lines).

Fig. 8 demonstrates that a very clear signal of differences in the redshift time drift between the timescape model and  $\Lambda$ CDM models might be determined at low redshifts when  $H_0^{-1} \frac{dz}{d\tau}$  should be positive. In particular, the magnitude of  $H_0^{-1} \frac{dz}{d\tau}$  is considerably smaller for the timescape model as compared to  $\Lambda$ CDM models. Observationally, however, it is expected that measurements will be best determined for sources in the Lyman  $\alpha$  forest in the range,  $2 < z < 5$ . At such redshifts the magnitude of the drift is somewhat more pronounced in the case of the  $\Lambda$ CDM models. For a source at  $z = 4$ , over a period of  $\delta\tau = 10$  years we would have  $\delta z = -3.3 \times 10^{-10}$  for the timescape model with  $f_{v0} = 0.762$  and  $H_0 = 61.7 \text{ km sec}^{-1} \text{ Mpc}^{-1}$ . By comparison, for a spatially flat  $\Lambda$ CDM model with  $H_0 = 70.5 \text{ km sec}^{-1} \text{ Mpc}^{-1}$  a source at  $z = 4$  would over ten years give  $\delta z = -4.7 \times 10^{-10}$  for  $(\Omega_{M0}, \Omega_{\Lambda0}) = (0.249, 0.751)$ , and  $\delta z = -7.0 \times 10^{-10}$  for  $(\Omega_{M0}, \Omega_{\Lambda0}) = (0.279, 0.721)$ .

## DARK MATTER AND THE TIMESCAPE SCENARIO

Since much of this conference has been about alternatives to standard nonbaryonic dark matter, I will briefly comment on the issue of dark matter vis-à-vis the timescape scenario. The timescape model only addresses large scale cosmological averages, and does not make specific predictions about dark matter typically inferred from direct observations of bound systems, such as rotation curves of galaxies, gravitational lensing or motions of galaxies within clusters. However, as has been discussed in ref. [2], in a re-examination of the post-Newtonian approximation within the averaging problem in cosmology, departures from the naïve Newtonian limit in an asymptotically flat space are to be expected. Consequently, any approach to treat galactic dynamics as a fully nonlinear problem within general relativity, such as the work discussed by Cooperstock at this conference [56, 57], is to be expected as potentially viable and compatible with the timescape scenario.

As discussed above, in the timescape scenario fits to supernova data, the BAO scale and the angular diameter distance of the sound horizon in CMB anisotropy data allow

one to estimate the dressed matter density parameter,  $\Omega_{M0}$ , while primordial nucleosynthesis bounds allow us to independently estimate the corresponding baryonic matter density parameter,  $\Omega_{B0}$ , and consequently of the nonbaryonic matter density parameter  $\Omega_{C0} \equiv \Omega_{M0} - \Omega_{B0}$ . As a result we find a mass ratio of nonbaryonic dark matter to baryonic matter of  $\Omega_{C0}/\Omega_{B0} = 3.1^{+2.5}_{-2.4}$ , with uncertainties from supernova data alone, or tighter bounds if constraints based on the angular diameter distance to the sound horizon are imposed. This potentially reduces the relative amount of nonbaryonic matter by a factor of two or more as compared to the standard concordance cosmology.

The Cooperstock–Tieu model [56, 57] demonstrates that the rotation curves of spiral galaxies can be reproduced<sup>9</sup> by a stationary axisymmetric rotating dust solution, obviating the need for spherical halos of dark matter as a required by the naïve use of Newtonian dynamics<sup>10</sup>. However, the Cooperstock–Tieu model does not specify the particle content of the dust. For the Milky Way the Cooperstock–Tieu mass estimate<sup>11</sup> of  $2.1 \times 10^{11} M_{\odot}$  [57] is a factor of 3–5 times *larger* than direct estimates of the combined baryonic mass of the galactic disk, bulge, bar and nucleus which are in the range<sup>12</sup>  $(4.2 - 7.2) \times 10^{10} M_{\odot}$  [61]. The ratio of the Cooperstock–Tieu mass to the observed baryonic mass of the Milky Way is thus in agreement with the global timescape estimate  $\Omega_{M0}/\Omega_{B0} = 4.1^{+2.5}_{-2.4}$ . Consequently, it is certainly possible for a significant amount of nonbaryonic dark matter to exist within the universe, reduced relative to the Newtonian dynamics estimate, if the Cooperstock–Tieu model, or something close to it<sup>13</sup>, operates at the galactic level.

The fact that several modified gravity approaches, including MOND [63], MOG [64], and conformal gravity [36], are able to phenomenologically reproduce various aspects of galactic and galaxy cluster dynamics to varying degrees of success, suggests that some simplifying principle remains to be found despite the amazing variety of structures described, which are far too complex to be modelled by simple exact dust solutions of general relativity such as those of refs. [56, 57]. Given a number of models which fit the same data [65], what is needed is that other falsifiable predictions of all the models

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<sup>9</sup> See ref. [58] for recent work which increases the number of galaxies whose rotation curves have been successfully fit by this model.

<sup>10</sup> A number of details of the Cooperstock–Tieu model have been disputed; see, e.g., ref. [59] for a summary. While the details are open to debate [57, 59], any deficiencies of the model might easily be an artefact of the simplifications of not including gas pressure, or other realistic features such as differentially rotating spiral arms and central bars. The basic premise of the Cooperstock–Tieu model, namely that nonlinearities in the Einstein equations can be important even in the weak field regime, stands as a consequence of general relativity that must be seriously considered at the galactic level.

<sup>11</sup> A recent Newtonian estimate of the mass of the Milky Way [60], including its dark halo, gives a mass in the range  $(5.7 - 10) \times 10^{11} M_{\odot}$  within a radius 80 kpc, or a total virial mass of  $(1.6 \pm 0.3) \times 10^{12} M_{\odot}$  at the virial radius  $R_{\text{vir}} = 300$  kpc. A direct numerical comparison with the Cooperstock–Tieu mass estimate is difficult, as the latter is confined to the mass within 30 kpc of the galaxy centre for which rotation curve data was available, and different density profiles can be assumed in the dark outer regions. However, by any measure the Cooperstock–Tieu mass estimate is certainly considerably reduced relative to the Newtonian estimate based on a dark matter halo.

<sup>12</sup> The central supermassive black hole, of mass  $\sim 4 \times 10^6 M_{\odot}$  is not included in this estimate of the Milky Way baryonic mass, as it is omitted in the Cooperstock–Tieu model.

<sup>13</sup> Another variant of the Cooperstock–Tieu model has been explored by Balasin and Grumiller [62].

are developed, which might rule some out. The standard Newtonian CDM hypothesis is at least so well developed that it leads to several testable predictions about structure formation; to the extent that it is arguably *already ruled out* by detailed observational studies of Local Group galaxies [66].

Although naïvely the timescape model appears to predict of order three times as much nonbaryonic matter as baryonic matter by mass, we should be careful to note that the ratio  $\Omega_{M0}/\Omega_{B0}$  measures the density of clumped matter at the present epoch relative to that of baryonic matter inferred from primordial nucleosynthesis, only when using solutions *averaged on cosmological scales*. Given the problems of defining gravitational energy in the absence of a timelike Killing vector or other exact symmetries, we should remain open to the possibility that the nature of clumped gravitational mass in cosmological averages is more than simply the sum of its particle constituents. Thus the difference,  $\Omega_{M0} - \Omega_{B0}$ , might not simply be nonbaryonic dark matter particles, but could include some component of gravitational energy that enters on some relevant scale of coarse-graining of bound systems, such as the transition from individual galaxies to galaxy clusters. Even for individual galaxies, such the Milky Way, the difference between the Cooperstock–Tieu mass estimate and the observed baryonic mass estimate could either simply be unaccounted dark matter (baryonic or nonbaryonic), or else include at least a partial contribution from gravitational energy that enters in the coarse-graining of the dust. Thus we should keep an open mind about the existence or nonexistence of nonbaryonic dark matter as long as these questions are not understood.

It is worth mentioning, however, that in the standard cosmology nonbaryonic dark matter is required to start structure formation going. At the surface of last scattering, dark matter density contrasts  $\delta\rho_C/\rho_C$  are expected to be an order of magnitude stronger than the baryonic density contrasts  $\delta\rho_B/\rho_B \sim 10^{-5}$ . The nonbaryonic dark matter overdensity contrasts provide the seed gravitational wells into which baryons fall. Although the relative amounts of nonbaryonic dark matter are reduced in the straightforward interpretation of the timescape scenario, there would be likely to be few changes to the basic qualitative scenario of the initiation of standard structure formation. If one wishes to completely eliminate nonbaryonic dark matter on the other hand, then one faces the formidable challenge of explaining how realistic structures can form from density contrasts which are only of order  $10^{-5}$  at last scattering. Thus the timescape scenario with the difference,  $\Omega_{M0} - \Omega_{B0}$ , interpreted as a nonbaryonic dark matter component remains the simplest scenario from the viewpoint of present understanding.

## DISCUSSION

Any serious physical theory should not only be founded on sound principles, but also provide predictions that can potentially rule it out. Much of the present review has therefore concentrated on several tests which might distinguish the timescape model from models of homogeneous dark energy. The (in)homogeneity test of Clarkson, Bassett and Lu is a definitive test independent of the timescape model with the potential to falsify the standard cosmology on large scales, since it tests the validity of the Friedmann equation directly. It would similarly rule out any modified gravity model which relied on a homogeneous geometry with a Friedmann–type equation at the largest scales.

In performing any tests, however, one must be very careful to ensure that data has not been reduced with built-in assumptions that use the Friedmann equation. For example, current estimates of the BAO scale, such as that of Percival *et al.* [46], do not determine  $D_V$  directly from redshift and angular diameter measures, but first perform a Fourier space transformation to a power spectrum, assuming a FLRW cosmology. Redoing such an analysis for the timescape model may involve a recalibration of relevant transfer functions.

In the case of supernovae, one must also take care since compilations such as the Union [52], Constitution [53] and Union2 [54] datasets use the SALT or SALT-II methods to calibrate light curves. In this approach empirical light curve parameters and cosmological parameters – *assuming the Friedmann equation* – are simultaneously fit by analytic marginalisation before the raw apparent magnitudes are recalibrated. As Hicken *et al.* discuss [53], a number of systematic discrepancies exist between data reduced by the different methods even within the  $\Lambda$ CDM model. In the case of the timescape model, we find considerable differences between the different approaches [27], which appear to be largely due to systematic issues in distinguishing reddening by host galaxy dust from an intrinsic colour variation in the supernovae. It is also crucial for the timescape scenario that data is cut at the scale of statistical homogeneity ( $z \sim 0.033$ ), below which a simple average Hubble law is not expected. For datasets reduced by the SALT or SALT-II methods there is generally Bayesian evidence that favours the  $\Lambda$ CDM model over the TS model. By contrast for datasets reduced by MLCS2k2 the Bayesian evidence favours the TS model over the  $\Lambda$ CDM model [27]. In principle, with perfect standard candles there are already enough supernovae to decide between the  $\Lambda$ CDM and timescape models on Bayesian evidence, but in practice one is led to different conclusions depending on how the data is reduced. It is therefore important that the systematic issues are unravelled.

The value of the dressed Hubble constant is also an observable quantity of considerable interest. A recent determination of  $H_0$  by Riess *et al.* [55] poses a challenge for the timescape model. However, it is a feature of the timescape model that a 17–22% variance in the apparent Hubble flow will exist on local scales below the scale of statistical homogeneity, and this may potentially complicate calibration of the cosmic distance ladder. Further quantification of the variance in the apparent Hubble flow in relationship to local cosmic structures would provide an interesting possibility for tests of the timescape cosmology for which there are no counterparts in the standard cosmology.

A huge amount of work remains to be done to develop the timescape scenario to the level of detail of the standard cosmology. At the mathematical level, we need to refine the notion of coarse-graining of dust in relation to the various scales of averaging, slicings by hypersurfaces in the evolution equations, and null cone averages. Whatever the outcome of such investigations, I find it exciting that much remains still to be explored in general relativity.

As long as the number of alternative theories is comparable to the number of “alternative” theorists, the detailed development of any alternative paradigm to the standard cosmology may take several years or decades, a timescale which also applies to the big science projects needed to perform precision observations such as redshift-time drift test [49]. Since one can always achieve better fits by adding new terms to relevant equations, every theorist is inevitably guided by intuition and aesthetic judgements about physical

principles as much as by existing observations and experiments.

My own theoretical prejudices are rooted in the knowledge that general relativity is on one hand an extremely successful theory of nature, in complete agreement with observations on the scale of stellar systems, and yet on the other hand, although it is based on deep physical principles, it is still also a theory which has not been completely understood in terms of the coarse-graining of dust, averaging, fitting, the statistical nature of gravitational energy and entropy, and the nature of Mach's principle. Although the nonlinearities of the Einstein equations may play a role in unravelling the mystery of dark matter [56, 57], my own opinion is that what is at stake is more than simply nonlinear mathematics, but also deep and subtle questions of physical principle.

Even if the retro-fit of a density distribution to observed galaxy rotation velocities via Einstein's equations [56, 57] could be independently shown to closely match the observed density distribution, there may be more subtle issues relating to coarse-graining and averaging which underlie the formation of the observed dust distributions, which may also be phenomenologically applicable to galaxies or galaxy clusters with less symmetry. It is worth noting that MOG [64] operates by an effective phenomenological variation of Newton's constant. Since direct observations never directly involve  $G$  but rather  $GM$ , my suspicion is that the phenomenology is pointing to the thorny issue of the definition of gravitational energy when averaging on different scales. This is the question we need to think more deeply about. The difficult problem of quasi-local gravitational energy in Einstein's theory may turn out not to simply be an arcane curiosity in mathematical relativity, but to be of direct importance for understanding the large scale structure of the universe.

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